The Problem of Volatility in the Black-Scholes Model

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Abstract

Since it was developed in 1973, the Black-Scholes model (BSM) has been the gold standard for option pricing. While more sophisticated and arguably more accurate models have been created since then, the BSM has been praised for its simplicity and ease-of-use. It achieves this simplicity due to a total of six assumptions it makes about the option in question, and while many of these assumptions are not accurate for the real world, they theoretically do not affect the accuracy of the formula too much. One assumption concerns the constant (as opposed to variable) nature of the volatility of the stock price. But since volatility is not able to be directly observed, it must be guessed at by using historical data. The goal of my research is to determine the best time frame of historical data to use to calculate volatility.

Introduction and Terminology

The Black-Scholes model (BSM) is used to give an accurate, fair price for what an option is worth. Since the BSM is the focus of this research, it is first necessary to understand what an option is, and to define some terminology surrounding these concepts. In the financial world, an option is a contract that allows the owner of the option to either buy or sell an underlying security associated with the option, which is known as exercising the option. For our purposes, this security is a stock, but it could also be a bond or some other asset. There are two types of options: put options and call options. A call option gives the option holder the right to buy the underlying stock, while a put option gives the option holder the right to sell the underlying stock. Note that the option holder is not obligated to do so, and they will only do so if it would bring in profit. We will only be dealing with call options, and the word "option" will now only refer to call options unless otherwise specified. However, the BSM does allow for the use of either type of option.

Another distinction that needs to be made is between American and European options. To understand this, one first needs to know that options do not last forever. Each option has an expiration date, and when this date is reached, the option can either be exercised or the option ceases to exist. An American option can be exercised at any point between when the option is created and when it expires. On the other hand, European options can only be exercised on the date that it expires. The BSM only deals with European options, so henceforth, any use of the word "option" implies that it is a European option unless otherwise specified.

When an option is created, a strike price is set for it. While the price of the underlying stock can most certainly change over time, the strike price is fixed and does not change over the lifetime of the option. If an option (remember, we are considering call options) has a strike price that is higher than the stock's market value, this is referred to as out of the money because exercising the option at this time would result in a loss. Similarly, in the money means that the strike price is lower than the market value and one could profit from exercising the option. If the strike price and the stock price are equal, this is called at the money.

The Black-Scholes Model

The BSM formula is as follows:

$$C(S_0,t)=S_0N(d_1)-Ke^{-r(t)}N(d_2).$$

In this formla, *C* is the price of the option. *K* is the strike price, which is the price at which you *exercise* the option, not the price you *buy* the option (this is the previous variable). The variable *t* is the time until expiration. N is the normal distribution, and d_1 and d_2 are as follows:

$$d_1 = \frac{\ln\left(\frac{S}{K}\right)+\left(r+\left(\frac{\sigma^2}{2}\right)\right)t}{\sigma\sqrt{t}}$$

$$d_2 = d_1 - \sigma\sqrt{t}$$

The BSM has six key assumptions about the stock in question and the market. Note that these are not accurate to what happens in the real world, but it makes the formula easy to use and theoretically does not affect accuracy too much—more on this later. First, the option is European, as stated above. Second, no dividends are paid out from the stock over the lifetime of the option. The formula can be modified to include dividends but that is beyond the scope of this paper. Third, markets cannot be predicted with 100%. Fourth, there are no transaction costs when creating the option. Fifth, the stock has normally distributed returns. And finally, the interest rate and volatility of the stock are constant and are known.

Volatility is an interesting variable in the BSM because it is the only variable that is not readily available unlike, for instance, the stock price or the time to expiration. This fact leads to two different uses of the BSM: one can either use the current option market price and back-solve for what is known as "implied volatility." Implied volatility is an educated guess about what the volatility will be across the lifetime of the option, not just what the volatility is at this moment. There are a host of issues present with implied volatility but we want to focus on the second use for the BSM.

The second use of the BSM is to use historical data about the stock price to calculate historical volatility, which is then used to calculate a fair price for the option. This raises the question of what time frame to use for historical volatility. There is no standard for how far back one should look at the stock prices in order to calculate volatility. Theoretically, historical volatility and implied volatility should be the same *if* the option is priced fairly.

My Research Procedure

Given that the BSM has some troubling assumptions concerning volatility, it would make sense to determine the degree to which this affects the usefulness of the formula. The first step was to find a database with historical data about the stock market and a variety of options. For this we used the Bloomberg Terminal at Florida Southern College. Unfortunately, due to issues with the terminal not always being functional, as well as circumstances with Covid-19, data from the terminal was never collected, so the rest of the procedure is theoretical. Even without these it quickly became apparent that collecting data for such a project as this was more complex than I had anticipated.

To begin collecting data, there needs to be a way to get a list of options that accurately represent the market at large. I had decided to use options from a random set of companies in the Nasdaq 100. Not many companies are needed, and the choice for how many is somewhat arbitrary, so I settled on 10. Next, from each company I needed to find a random selection of options. These options needed to be from different time periods because I did not want market factors (such a market crash) to affect volatility as much as possible. This is difficult because there is no particular pattern to when options are created. It is quite easy to find a list of all current options but having all the options be from one time period is problematic as already stated. It is also quite easy to look up all options created on a particular date, but it is impossible to know when options are created so this is not very useful. Ultimately, searching for options created on every possible date in a particular time frame may be the only possible way to make this type of research work, despite how extremely tedious this would be. Once the options have been found, it is now a matter of analyzing them.

Implied volatility can be easily calculated from data collected from the Bloomberg Terminal by inputting all the correct values into the BSM. Historical volatility for the past X days is also able to be extracted automatically from the Bloomberg Terminal. I chose 30, 60, and 90 days as the comparison points.

Admission of Flaws

It is only fair to recognize some of the downfalls and biases that are present in any research endeavor. Aside from the issues listed above concerning collection of the data, one large problem with this research is the ubiquitous nature of the BSM. Since a great many investors choose to use the BSM as a part of their investing strategy, it skews the data to make the BSM look as if it is more useful/accurate that it really is. It is impossible to know where and how the BSM is being used by investors, however, so there is little that can be done to fix this.

Another flaw of this research is the fact that option in the real world may not always be priced accurately. As stated previously, implied volatility and historical volatility only line up when the option is priced “fairly.” If they are not, then analysis on what time frame to use for historical volatility does not make much sense. It is difficult to say how often options are priced fairly, and to what degree they differ from their fair pricing.

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