The Travelling Salesman

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Abstract

The traveling salesman problem (TSP) involves trying to find the optimal path to tour a collection of cities, or points on a graph, based on the distance between them. The goal of our project was to find a heuristic, an approximated solution, to find the shortest route possible.

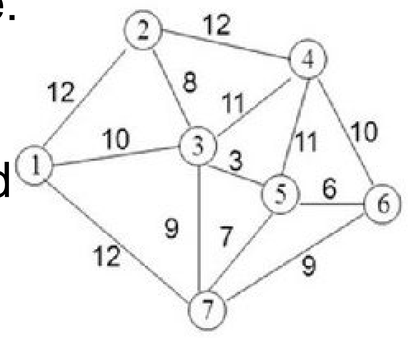
Introduction

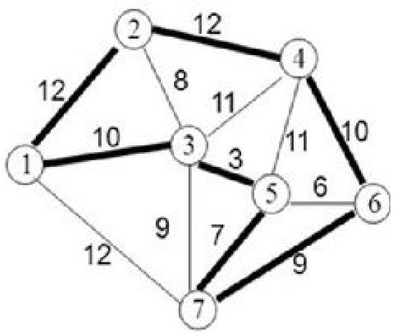
TSP is among the most famous CS problems categorized as NP hard, which means fully solving the problem takes an exponential amount of time. This is why heuristics are used, to approximate the solution, in an acceptable amount of time. We used Christofide's algorithm with a 2-opt improvement on top of that, guaranteeing, at worst, that our path will be 1.5x the optimal path.

Methods

We began by researching the different algorithms and heuristics for the TSP, and ended up on Christofide's for nonmetric graphs, as it gave a worst case of 1.5x the optimal length, where all the others gave 2x.

- Create a minimum spanning tree (MST) from the graph (we used Prim's algorithm)
- Duplicate the edges of the vertices, making them directional and of even degree
- Create a Eulerian circuit over the new combined MST and edges to approximate a path
- Perform a 2-opt optimization to improve our path





Results

The overall Big-O runtime of the non-metric algorithm is $O(V \cdot E^3)$. The final path provided by our algorithm is at worst 1.5x the optimal TSP path, based on our MST construction and 2-opt tour improvement.

Discussion

Our implementation of our picked heuristic and algorithms were successful. While we would like to have a more optimal path, 1.5x is the best that can be done now in a reasonable time. Aside from the success of the program, we learned a lot about the dynamics of team development, which has given a lot of insight and helped us professionally. Overall, it was a positive and essential experience as a CS major.

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